Equilibrium β-limit in a quasi-axisymmetric stellarator with self-consistent bootstrap current

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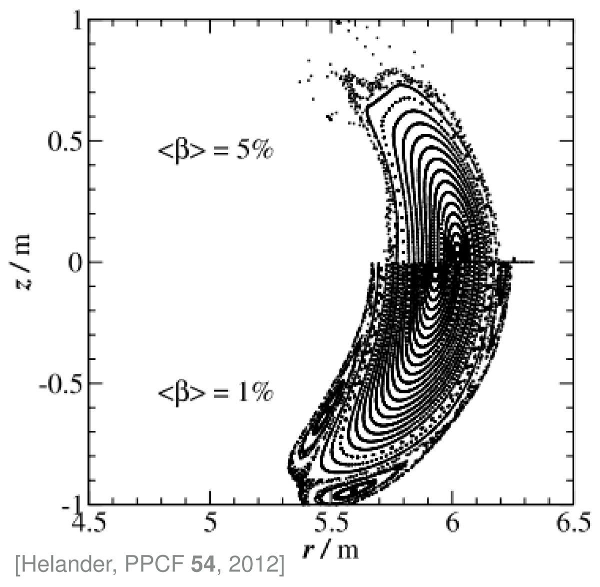
Motivation

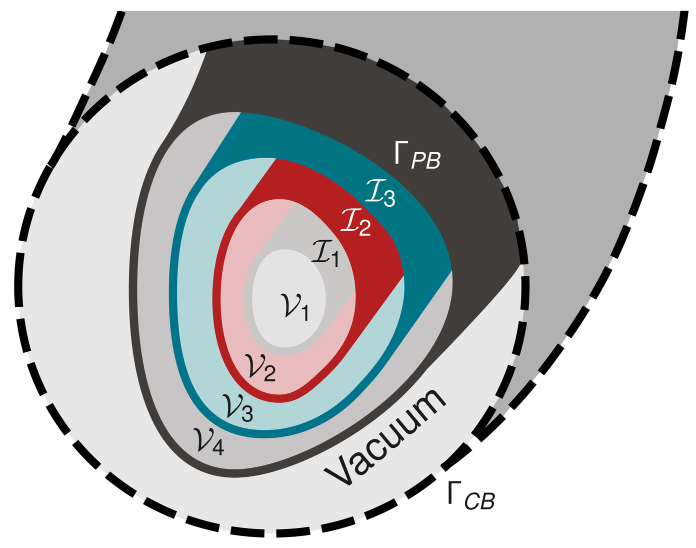


[Helander, PPCF **54**, 2012]

- The β-**limit(s)** in stellarators can be set by either stability or **equilibrium properties**
- Bootstrap current is the main source of current in stellarators without external current drives
- Including the bootstrap current can be of paramount importance for equilibrium calculations
- **Aim:** (I) Compute SPEC equilibria with self-consistent bootstrap current, and (ii) understand field line topology dependencies on pressure

The stepped-pressure equilibrium code (SPEC)

- SPEC [1] computes free-boundary, multi-region, relaxed magnetohydrodynamic equilibria
- **Magnetic field line topology** is contrained on a finite number of ideal nested surfaces $\mathcal{I}_l$
- **It defines N_{vol} nested volumes** $\mathcal{V}_l$, bounded by the plasma boundary Γ_{PB}
- The **computational boundary** Γ_{CB} surrounds the plasma and is within the coils



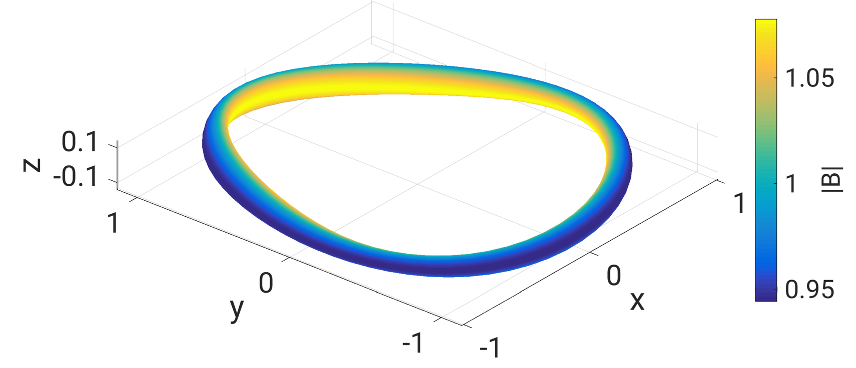
- Solution is a **force-free field** satisfying a jump condition across $\mathcal{I}_l$

$$\text{In } \mathcal{V}_l: \quad \nabla\times\mathbf{B} = \mu_l\mathbf{B}$$

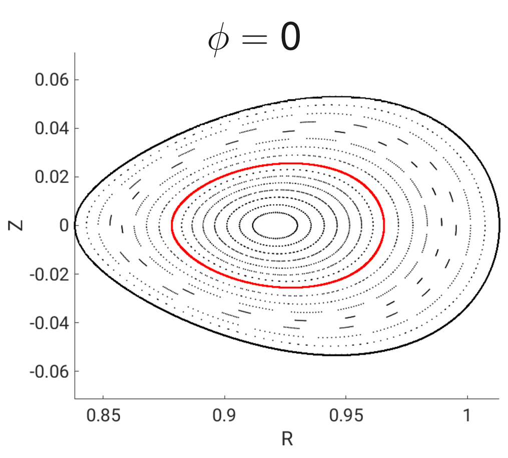
$$\text{At } \mathcal{I}_l: \quad \left[\left[p+\frac{B^2}{2\mu_0}\right]\right]_l = 0$$

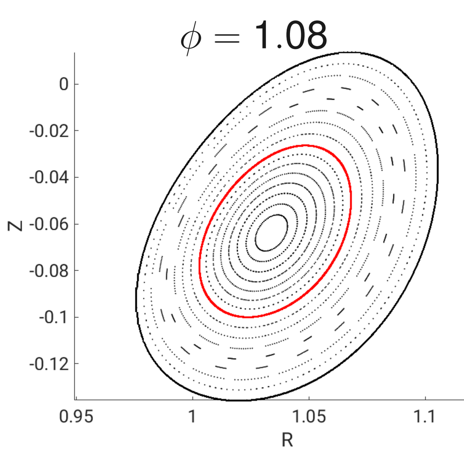
- Solution is determined by the pressure and two additional profiles (constraints), for example the net toroidal current in $\mathcal{V}_l$, and at $\mathcal{I}_l$, denoted $I^v_{\phi,l}$ and $I^s_{\phi,l}$ respectively [2]

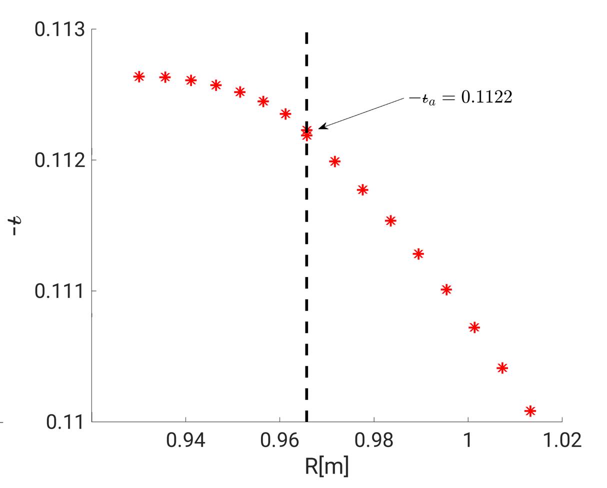
Quasi-axisymmetric (QA) configuration



- We consider a two-field periods QA configuration with aspect ratio $R_0/a = 30$ (courtesy of R. Nies)
- Single plasma volume, $N_{vol} = 1$
- We set $I^v_{\phi,1} = 0$
- The net toroidal current at the plasma edge, $I^s_{\phi,1}$, is as-of-yet undetermined







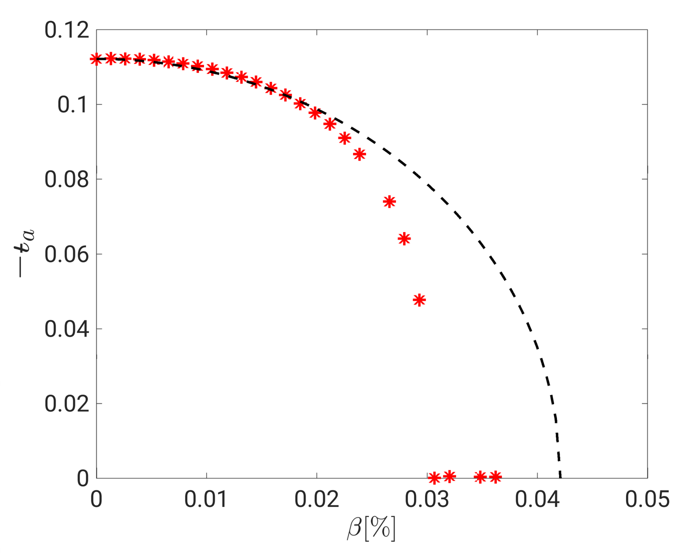
Current-free scan

- Set $I^v_{\phi,1} = I^s_{\phi,1} = 0$ for all pressure
- Rotational transform at the plasma edge, ι_a, **decreases with** β, until reaching $\iota_a = 0$
- SPEC calculations lead to $\beta_{lim} = 0.03\%$
- High Beta Stellarator (HBS) expansion [3] gives

$$\iota_a = \iota_V\sqrt{1-\frac{\beta^2}{\epsilon_a^2\iota_V^4}} \tag{1}$$

- Solving for $\iota_a = 0$ leads to

$$\beta_{lim} = \epsilon_a\iota_V^2 = 0.04\% \tag{2}$$



- A small equilibrium β-limit is expected because of the small vacuum rotational transform
- Equilibrium β-limit is **lower** than in an **equivalent classical stellarator**
- **Shaping** is **detrimental** to the field line topology in this particular configuration

Self-consistent bootstrap current

- Landreman *et al.* [4] implemented the "Redl formulas" [5] in SIMSOPT [6],

$$\langle\mathbf{J}\cdot\mathbf{B}\rangle^{\text{Redl}} = -\frac{G}{\iota}\left(\mathcal{L}_{31}\left[n_eT_e\frac{d\ln n_e}{d\psi_t}+n_iT_i\frac{d\ln n_i}{d\psi_t}\right] + p_e(\mathcal{L}_{31}+\mathcal{L}_{32})\frac{d\ln T_e}{d\psi_t}+p_i(\mathcal{L}_{31}+\mathcal{L}_{34}\alpha)\frac{d\ln T_i}{d\psi_t}\right) \tag{3}$$

- Assuming stepped density and temperature profiles,

$$\langle\mathbf{J}\cdot\mathbf{B}\rangle^{\text{Redl}}_{l,\pm} = -\frac{G_l^\pm}{\iota_l^\pm}\left(\mathcal{L}^\pm_{31,l}\left[T^\pm_{e,l}[[n_e]]_l+T^\pm_{i,l}[[n_i]]_l\right] + n^\pm_{e,l}(\mathcal{L}^\pm_{31,l}+\mathcal{L}^\pm_{32,l})[[T_e]]_l+n^\pm_{i,l}(\mathcal{L}^\pm_{31,l}+\mathcal{L}^\pm_{34,l}\alpha^\pm_l)[[T_i]]_l\right)\delta(\psi_t-\psi_{t,l}) \tag{4}$$

- SPEC has been coupled to booz_xform [7] to evaluate all blue coefficients
- From a SPEC equilibrium, one can evaluate

$$\langle\mathbf{J}\cdot\mathbf{B}\rangle^{\text{SPEC}}_{l,\pm} = \frac{1}{V'(\psi_{t,l})}\iint\frac{dS}{|\nabla\psi_t|}\left\{(\hat{\mathbf{n}}\times[[\mathbf{B}]]_l)\cdot\mathbf{B}^\pm_l\delta(\mathbf{x}-\mathbf{x}_l)\right\} \tag{5}$$

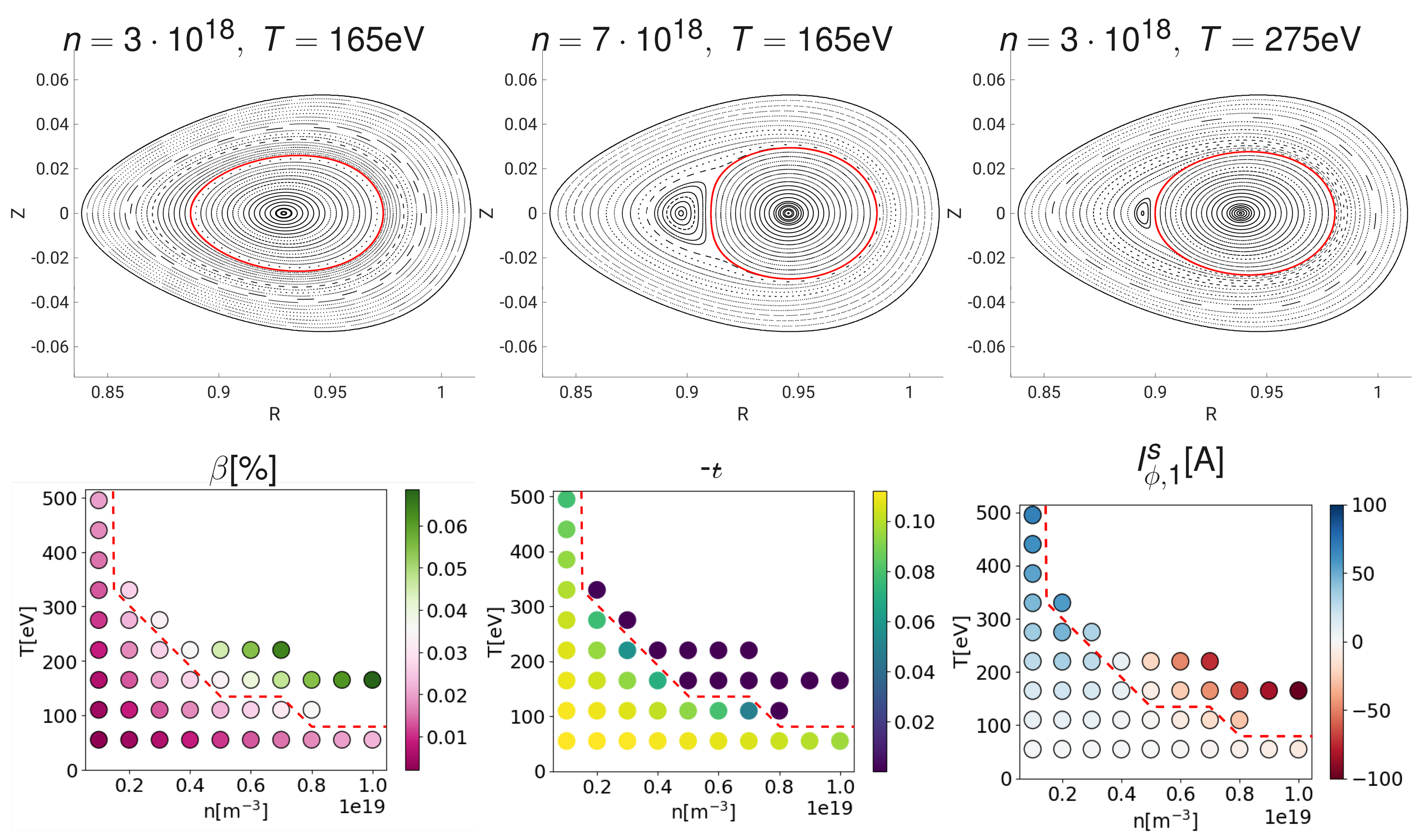
$$= \frac{1}{V'(\psi_{t,l})}\iint d\theta d\phi\left\{\left([[B_\theta]]_lB^\pm_{l,\phi}-[[B_\phi]]_lB^\pm_{l,\theta}\right)\delta(\psi_t-\psi_{t,l})\right\} \tag{6}$$

- An **optimization target function** is constructed to enforce self-consistent bootstrap current

$$f_{\text{bootstrap}} = \left(\langle\mathbf{J}\cdot\mathbf{B}\rangle^{\text{Redl}}_l-\langle\mathbf{J}\cdot\mathbf{B}\rangle^{\text{SPEC}}_l\right)^2 \tag{7}$$

- The degree of freedom is the net toroidal current $I^s_{\phi,1}$ located Γ_{PB}
- We enforce **no external current** drive, $I^v_{\phi,1} = 0$
- Optimization is driven by SIMSOPT

Density and temperature scans



- An **ideal equilibrium** β-**limit** is found, where a large $(m,n)=(1,0)$ island opens
- Similar β-limit is observed in a classical stellarator with small bootstrap current [8]
- Equilibrium β-limit is small ($\sim 0.03\%$)
- Including the effect of the **bootstrap current has little to no effect** on the ideal equilibrium β-limit for this configuration

Conclusions & Outlooks

- **SPEC equilibria with self-consistent bootstrap current** were computed for the first time
- Free-boundary QA equilibria were calculated for a wide range of densities and temperatures
- An **ideal equilibrium** β-**limit** is hit at $\beta\sim 0.03\%$
- Similar studies in a smaller aspect ratio QA configuration with larger rotational transform should be considered
- The sensitivity of field line topologies to pressure variation can now be compared in various quasi-symmetric configurations
- Verification of the implementation is in progress

Acknowledgements

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